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Short Communications

Singular Value Decomposition Based Sub-band Decomposition and Multi-resolution (SVD-SBD-MRR) Representation of Digital Colour Images

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ABSTRACT

Contemporary digital image processing applications require a suitable tool for further applications and processing. In this paper, a Singular Value Decomposition (SVD) based new sub-band decomposition and multi-resolution representation of digital colour images is proposed. Simulation was performed using MATLAB on Lenna image and the resultant sub-bands, which contained different directional details, are shown. Through a quantitative analysis, it is justified that the proposed method is better than other contemporary methods as it is suitable for compression and other image processing applications.

Keywords: Image representation, subband decomposition, multiresolution representation, compression

LIST OF ABBREVIATIONS

DCT	:	Discrete Cosine Transform
DWT	:	Discrete Wavelet Transform
JPEG	:	Joint Picture Expert Group
KLT	:	Karhunen-Loeve Transform
MRR	:	Multi-Resolution Representation
SBD	:	Subband Decomposition
SPIHT	:	Set Partitioning in Hierarchical Tree
SVD	:	Singular Value Decomposition

INTRODUCTION

Contemporary scientific and engineering applications require a lot of digital signal, image, and multimedia data processing. This processing requires suitable tools for signal analysis for different signal and image processing applications. An image can be analyzed by decomposing it into various

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sub-bands which contain high and low frequency information. Wavelet sub-band decomposition suggested by Mallat (1989) is a tool for analyzing two dimensional image signals. The sub-band decomposition is a process of decomposing a signal along the horizontal, vertical, and diagonal directions by maintaining a constant number of pixels required to represent the image (Antonini *et al*., 1992).

Any image transformed which is suitable for sub-band decomposition should have the property of representing the original signal as linear combination of elementary atoms of transform, where each and every atom is a structured expansion having fixed localizations producing the tilling effect in Space-Frequency (SF) planes. According to Ramchandran *et al.* (1993), the case when bases are adaptive or signal dependent is the case of the best bases giving the SF localization gathering most of the signal energy in the fewest possible number of coefficients and in this case, no priori assumption of the signal is required. Hence, it is possible to directly design multi-channel filter banks. In principle, it is similar to a two-channel case using more involved analysis and design process.

Ramchandran & Vettreli (1993) and Ramchandran *et al*. (1996) reported that Karhunen-Loeve Transform (KLT) and Discrete Cosine Transform (DCT) are the examples of N-Channel filter banks, restricting the length of filters by the block size of N, corresponding to the sub-sampling factor. The use of fixed transform bases and model-based quantization strategies were employed in the traditional image coding models (JPEG, JPEG2000, etc., using either DCT or DWT). These image coding algorithms are useful for little class of signals or data, which are stationary in nature (signal distribution statistics is constant with respect to time or space), whereas image signals are of non-stationary in nature (Antonini *et al*., 1992; Ramachandran & Vettreli, 1993; Ramachandran *et al*., 1996). For the non-stationary signals or data, the fixed transforms coding are insufficient to be used and for dealing with a large class of signals of unknown statistics or time or space variability signals, and thus, the adaptive image transform approach is more suitable and robust.

The multi-resolution decomposition provides a mean to have a scale invariant representation of any image signal or data. A suitable multi-resolution representation should have a constant interpretation property with changing scale. At different resolutions, different physical structures of the image may be represented. Larger structures are represented by coarse resolution, whereas fine structures are represented by higher resolutions. The wavelet multi-resolution representation by Mallat (1989) is a very popular tool for different image processing applications like analysis, enhancement, watermarking and compression, etc. The existing multi-resolution techniques have the disadvantage of fixed transform bases. Meanwhile, the SVD has an optimal decorrelation, sub-rank approximation property. It is the signal or image dependent adaptive transform which gives a new choice for the SVD-Subband (SVD-SBD) and Multiresolution Representation. Kakarala & Ogunbona (2001) proposed the only available block-based SVD-Multiresolution Representation (SVD-MRR) for 2D-Signals. This algorithm was based on image sub-block decomposition and the rearrangement of singular values and one of the singular vectors of these image sub-blocks. For N x N image sub-block, the N^4 components are required for reconstruction; hence, more number of coefficients than that of the original image has to be coded to give the cause for the non-suitability for compression by using the algorithm proposed by Kakarala & Ogunbona (2001). In this paper, the sub-band decomposition and the multi-resolution form of SVD have been proposed while maintaining the properties of Singular Value Decomposition.

SINGULAR VALUE DECOMPOSITION

The SVD is a linear orthogonal transform and it is suitable for digital image processing applications due to its high energy compaction efficiency for a given block of image data. For a given image block I of size N x N, the singular vectors u_i & v_i are known as the Eigen vectors of II^T and I^TI, respectively. The singular value s_i is the square root of the Eigen values of II^T and I^TI. The image block can be represented as:

$$I = USV^T \tag{1}$$

Where, U, S & V are the N x N matrices containing the orthogonal column vectors (Eigen vectors u_i & v_i) respectively, and S is a diagonal matrix with the singular values along the main diagonal:

$$S = \begin{bmatrix} s_1 & 0 & . & 0 \\ 0 & s_2 & . & 0 \\ . & . & . & 0 \\ 0 & 0 & 0 & s_N \end{bmatrix}_{N\times N} = \begin{bmatrix} \sigma_1^2 & 0 & . & 0 \\ 0 & \sigma_2^2 & . & 0 \\ . & . & . & 0 \\ 0 & 0 & 0 & \sigma_N^2 \end{bmatrix}_{N\times N} \tag{2}$$

where, $s_1 \geq s_2 \geq ...... \geq s_N \simeq 0$ and $\sigma_1^2 \geq \sigma_2^2 \geq ...... \geq \sigma_N^2 \simeq 0$

k is the rank of the matrix I, where $k \leq N$. Hence, the approximated image $\overline{I}$ can be given as:

$$\overline{I} = U_k S_k V_k^T = USV^T = I \tag{3}$$

In order to achieve the goal of compression, the following rank should be used:

$$k \leq \frac{N \times N}{(1+2N)} \tag{4}$$

The original image can be estimated using the low rank approximation while retaining only R, the largest Eigen values and corresponding Eigen vectors, as follows:

$$\hat{I} = \sum_{i=1}^{R} u_i s_i v_i^T \tag{5}$$

Where, $R \leq k \leq N$ and the discarded Eigen values are $\sigma_{R+1}^2, \sigma_{R+2}^2, \sigma_{R+3}^2 ....,, \sigma_N^2$. Thus, the square error after discarding (k - R) lower Eigen values, that is simply truncation error, is given by the following equation:

$$\sum_{i=1}^{N} \sum_{j=1}^{n} \left| I(i,j) - \hat{I}(i,j) \right|^2 = \sum_{m=R+1}^{N} \sigma_m^2 \tag{6}$$

$$MSE = \left(\frac{\sum_{m=R+1}^{N} \sigma_m^2}{N \times N} \right) \tag{7}$$

Motivation for SVD-SBD: Let X be a 2x2 image sub-block and its SVD is taken to form three matrices as follows:

$$X = svd(X) = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} S_{11} & 0 \\ 0 & S_{22} \end{bmatrix} \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \quad (8)$$

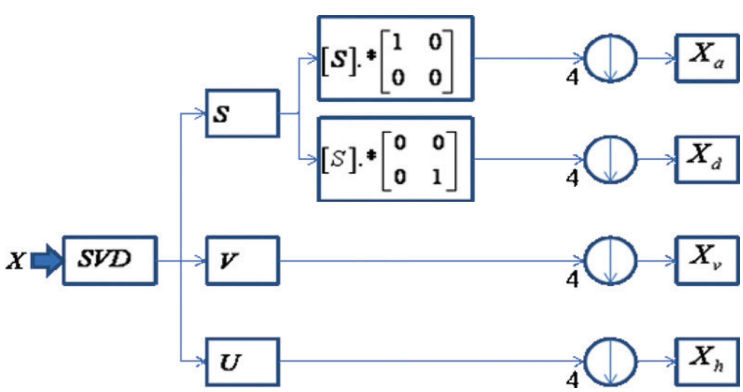
From [8], U_{11}, V_{11} and S_{11} are responsible for horizontal, vertical, diagonal details, and the approximation of the image sub-block X respectively. The presences of details are quantified as shown in Table 1 below.

TABLE 1
The quantification of the details present in the image sub-block

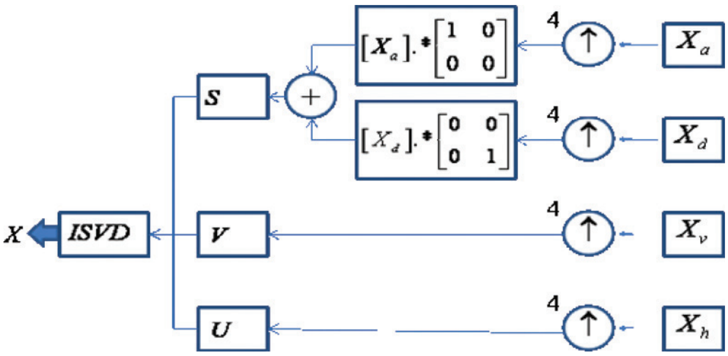
S_{11}	U_{11}	V_{11}	Detail present
$S_{11} = 0$	$U_{11} \neq -\frac{1}{\sqrt{2}}$	$V_{11} = -\frac{1}{\sqrt{2}}$	Horizontal details
$S_{11} = 0$	$U_{11} = -\frac{1}{\sqrt{2}}$	$V_{11} \neq -\frac{1}{\sqrt{2}}$	Vertical details
$S_{11} \neq 0$	$U_{11} = -\frac{1}{\sqrt{2}}$	$V_{11} = -\frac{1}{\sqrt{2}}$	Diagonal details
$S_{11} \neq 0$	$U_{11} \neq -\frac{1}{\sqrt{2}}$	$V_{11} \neq -\frac{1}{\sqrt{2}}$	All details

2D-SVD-SBD STRUCTURE

2D-Singular Value Decomposition based Subband decomposition and reconstruction architectures are given in *Figs. 1(a)* and *(b)*. The procedure shown in *Fig. 1(a)* was recursively applied to all the image sub-blocks to get different image sub-bands.



1(a)



1(b)

Fig. 1(a): SVD-Subband Decomposition; (b) SVD-Subband Reconstruction

RESULTS

Figs. 2(a) and *(b)* show the 1st and 2nd levels of 2D-SVD-SBD formation for the Lenna test image. The top left part of the image gives the horizontal detail, the bottom left gives the vertical detail, the bottom right gives the diagonal details, and the top right shows the approximation of the original image. Also the reconstructed image is shown in *Fig. 3*.

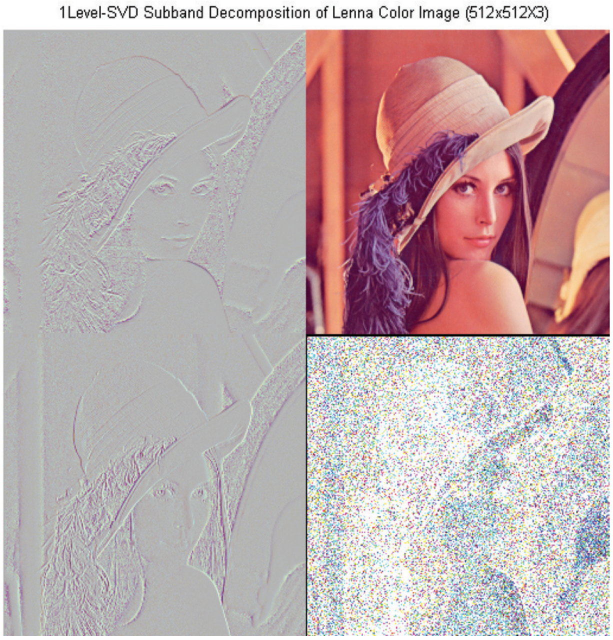


Fig. 2(a): 1st Level SVD-SBD of Lenna, Top right: approximation, Top left: horizontal details, Bottom left: vertical details and Bottom right: diagonal details

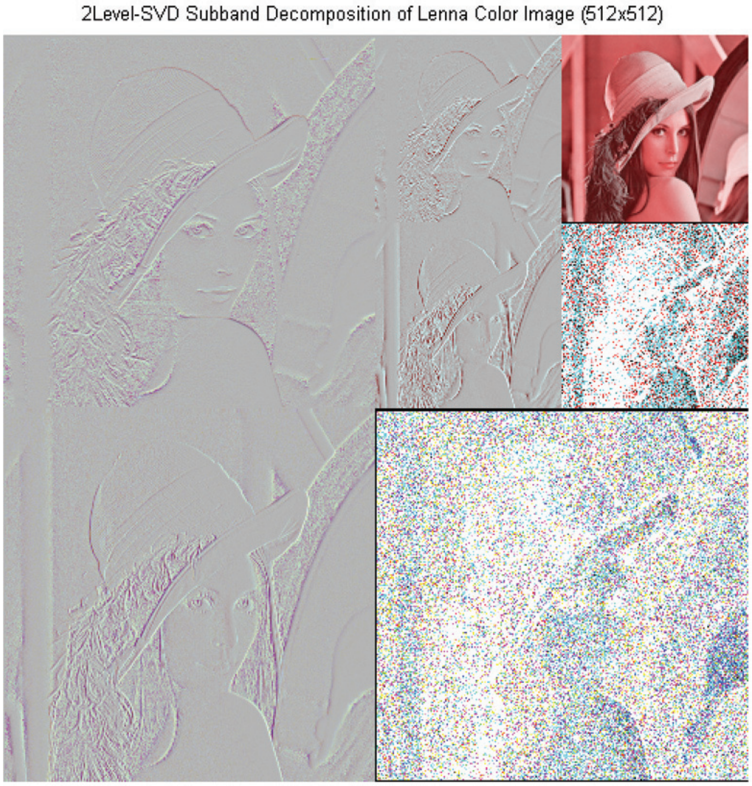


Fig. 2(a): 2nd Level SVD-SBD of Lenna

Fig. 3: Reconstructed Lenna image

CONCLUSIONS

In this paper, a new choice of sub-band decomposition has been presented for different image processing applications. This particular method utilizes the maximum energy compaction which is not possible in the case of wavelet sub-band decomposition. This method was also compared with the method suggested by Kakrala & Ogunbuna (2001), with respect to the number of coefficients required for the reconstruction of N x N image and N^2 rather than N^4 which can therefore be used for compression as well. This method may provide new dimensions to digital image compression. In addition, the complexity of the proposed algorithm is lower than the contemporary ones. Meanwhile, other applications of SVD-SBD can be exploited in future.

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